MULTIAXIAL FATIGUE LIMIT EVALUATION BY CRITERIA BASED ON VON MISES STRESS

Jan Papuga[a]

[a] Fac. of Mech. Engng., Czech Technical University in Prague, Prague, Technicka 4, 166 07 Prague 6, Czech Republic

Abstract The paper focuses on the evaluation of the set of experiments gathered in the FatLim database (www.pragtic.com/fatlim.php). The experimental results there are fatigue limits under multiaxial or uniaxial loading including mean loads. A set of carefully selected 407 experiments is used as an input for analyses of validity of various criteria based on von Mises stress. Though the trend today tends to more complex multiaxial fatigue analyses and there are numerous criteria proposed for such a solution, Mises stress based criteria are very simple and efficient for use. Their use in the engineering practice is still very substantial. Therefore, in addition to the previous work on multiaxial analyses, the author focuses on these simple analytical tools (as Manson-McKnight, Sines, etc.) and analyses their applicability for various sets of material classes or experimental conditions. The comparison with more enhanced multiaxial methods is also provided.

1 INTRODUCTION

The trend today tends to more complex multiaxial fatigue analyses and there are numerous criteria proposed for such a solution. These criteria are often based on an evaluation of the stress state on a set of separate planes. Logically, in comparison to common uniaxial calculation procedures, the solution is more complicated and costs more computational effort. On the other hand, criteria based on Mises stress are very simple and efficient for any immediate use. In comparison to multiaxial methods, the computational procedure is so simple, that the engineers are able to implement them by themselves for an automated evaluation of FEA-results. Therefore, in addition to the previous work on multiaxial criteria, the focus is set now on these simple analytical tools. Various methods based on Mises stress are presented and their preditictive capability analysed on a large experimental set of fatigue limits under multiaxial and mean stress affected uniaxial loads. In addition to their mutual comparison under various load conditions (in-phase, out-of-phase, with/without mean stresses, …) these criteria compared also with the most prominent multiaxial criteria.

2 TEST SET

The prediction methods are compared here only as regards their capability to estimate the fatigue limit value under complex load conditions – be it either multi-channel loading or simple uniaxial loading with non-zero mean stress. The test set is extracted from the experimental results gathered in the FatLim database (accessible for free on http://www.pragtic.com/experiments.php) and is described and used also e.g. by Papuga [1]. The experiments cover only unnotched specimens.

During their derivation from full FatLim database, the experimental data points with poor statistic substantiation, poor material data, negative mean stresses and different load frequencies on different load channels were rejected, as is noted by Papuga in [1]. The main reason for removing the experiments with negative mean stresses was the reasoning that the multiaxial criteria evaluated there are not well prepared for such load conditions and the only mechanism included is zeroizing the negative mean stress during the computation. This solution logically leads to more conservative results, which is acceptable but it could hide behaviour of these methods under certain conditions.

At the comparison presented here nevertheless, two methods provide the same solution, unless the experiments with negative mean stresses are also admitted. Because we are looking for the simplified solution and do not place impact so much on the response to the non-proportional multiaxial loading, the negative mean stresses were also admitted to the analyses for these two methods. It means explicitly, that in addition to 407 experiments evaluated by Papuga in [1], the following experiments from FatLim database are included: BaB09, BaB10, Bai13, Bai14, HeG05, HeG06, Hei07, HZ03, HZ04, HZ05, Mi01, Mi03, SiB01, SiB03, SiB04, thus forming an alternative test set of 422 experiments, which is referred to in Sec. 5.1.8.

3 EVALUATION

The experimental data used for validation of various presented methods relates to multiaxial loading at fatigue limit. The application of the calculation procedures mentioned below results in the equivalent stress amplitude $\sigma_{a,eq}$ that has to be equal to the fatigue limit under fully reversed axial loading f_{-1}.
The prediction quality was analysed using the fatigue index error ΔFI as a parameter. This parameter represents the relative deviation of an equivalent load $\sigma_{a,eq}$ at the multiaxial fatigue limit from the uniaxial fatigue limit in fully reversed axial loading f_{-1}:

$$\Delta FI = \left(\sigma_{a,eq} - f_{-1}\right) / f_{-1} \cdot 100\% \qquad (1)$$

According to this relation, values of ΔFI below zero indicate that the criterion does not predict the failure though it should and the results will be referred as "non-conservative" or "unsafe", on the other hand, positive values of ΔFI will be referred as "conservative" or "safe".

4 CHECKED METHODS

The paper is focused on the evaluation of simple methods in cases where the loading is more complex than simple reversed axial loading. Anyhow, results of three criteria representing the classic multiaxial approaches based on a critical plane evaluation or enclosing the load path in the Ilyushin's deviatoric space are presented as well, so that they could show the assets of such solutions.

4.1 Multiaxial solutions

The most classic multiaxial solution is based on an assumption that the material along some plane encounters such a great local loading (shear and normal stresses) that the crack is formed. In the case of the high-cycle fatigue, the initiation period takes a substantial part of the complete lifetime and the relation between crack initiated on such a plane and the damage of the component can be accepted. The multiaxial methods use usually complex damage parameters with superposed normal and shear stresses, because it was found that neither sole shear nor normal stresses provide acceptable prediction quality. Anyway, the necessity to use an algorithm that scans different positions of candidate planes in order to find the critical one causes substantial burden as regards time and computational force.
Another potential solution is to evaluate the damage parameter related to individual planes in such a way, that the damage parameters over all possible planes are integrated together. Papuga [1] mentions increased computational effort in comparison to the critical plane approaches and notes that the results of integral methods checked by him had not been persuasive enough to accept such a prolongation in the evaluation.
The last major group of solutions uses evaluating of the load path in the five-dimensional Ilyushin's deviatoric space. Usually, some geometric entity enclosing the load path data points is looked for. Even here, the search for the complete solution takes more time than is usual for simple uniaxial methods. A comprehensive comparison of methods of these 3 types was presented in the paper [1].

4.1.1 Dang Van

From all the methods evaluated in [1], the Dang Van method [2] is the most often accepted solution as regards the number of occurrences in commercial fatigue solvers (MSC.Fatigue, nCode Desing Life, LMS Virtual.Lab component Durability, Fe-Safe, Fearce – only FemFat fatigue solver does not have this method implemented). Thus it serves here as a perfect candidate for comparison with simplified methods. It formula holds:

$$\sigma_{a,eq} = a_{DV} \cdot C_a + b_{DV} \cdot I_{1,\max} \qquad (2)$$

Here the amplitude of the damage parameter on the left hand side is equal to the linear combination of the maximum shear stress amplitude C_a encountered on any plane and of the maximum value of the first invariant of the stress tensor $I_{1,max}$ during the cycle.
The scatter of results of the method [1] evaluated on the FatLim data set can be mostly attributed to the problems with mean stress effect incorporation, where pure torsion loading tends to too non-conservative results, while biaxial loading (axial and axial loadings in two perpendicular directions) are processed with results too conservative. In addition to this failure related to the mean stress effect, Papuga [1] finds a distinct change of the criterion behaviour (manifested by the mean value of the fatigue index error) for in-phase and out-of-phase loading without any mean stresses in both cases.

4.1.2 Sines

Papadopoulos et al in [3] provided an enhanced solution for the Sines [4] method, using the minimum hyper-ball circumscribed to the load path in the Ilyushin deviatoric space and equalled it to the square root of the amplitude of the second invariant of the stress tensor. The deviatoric space is defined by five components derived from the stress tensor deviator:

$$s_1 = \sqrt{\frac{3}{2}} s_{xx}; \qquad s_2 = \frac{1}{\sqrt{2}}\left(s_{yy} - s_{zz}\right); \qquad (3)$$
$$s_3 = \sqrt{2} s_{xy}; \quad s_4 = \sqrt{2} s_{xz}; \quad s_5 = \sqrt{2} s_{yz}.$$

The criterion itself follows this formula:

$$\sigma_{a,eq} = a_{DV} \cdot \sqrt{J_{2,a}} + b_{DV} \cdot I_{1,m} \qquad (4)$$

Though the analysis of the minimization problem for finding the minimum enclosing hyper-ball is not a simple one, the Sines criterion has the asset that it is run only once for the evaluated load path. Thus the solution is much more straightforward and the computation quicker, if compared to the Dang Van method.

There is a very similar solution proposed by Crossland [5]. The formula is nearly identical to Eq. (4) with the exception of the maximum value of the first invariant at its end instead of the mean value used by Sines. Actually, Papuga in [1] classifies the Crossland solution as much more accurate than the Sines method, though the scatter of prediction results marked by the high standard deviation is high. Anyway, because of the referenced paper [7], from which the methods presented here were excerpted, refer to Sines, this method was used as the right one for comparison purposes.

4.1.3 Papuga PCr

Papuga PCr method was first presented in [6] and is included into the test set because of its best prediction results as regards the range and standard deviation of the fatigue index error. The method is a hybrid combination of the squared shear stress amplitude and a linear form of normal stress further enhanced by the established mean stress effect:

$$\sigma_{a,eq} = \sqrt{a_{PC} \cdot C_a^2 + b_{PC} \cdot \left(N_a + d_{PC} \cdot N_m\right)} \qquad (4)$$

Probably the biggest asset in comparison to other methods compared by Papuga in [1] is the mean stress effect inclusion, which is closer to real experimental data.

4.2 Solutions based on von Mises stress

The methods presented in the further text are mainly excerpted from the NASALIFE report [7]. The common computation scheme follows the separation to an amplitude and a mean value. In such a case the use of the Walker correction factor for mean stress is proposed [7]:

$$\sigma_{a,eq} = \sigma_a^{\gamma} \cdot \left(\sigma_m + \sigma_a\right)^{1-\gamma} \qquad (5)$$

Walker mean stress correction gave better results in comparison to other corrections (Goodman, Gerber,…), when tested by us. Usually, the value of γ parameter has to be derived by fitting to the experimental data. Thanks to the experimental data sets from FatLim database used here, the information of the fatigue limit in repeated axial loading f_0 could be used for the computation of γ by transforming Eq. (5) to:

$$\gamma = 1 - \log\left(2 \cdot f_{-1} / f_0\right) / \log(2) \qquad (6)$$

4.2.1 Manson - McKnight

There are at least several formulations of the Manson-McKnight criterion. The basic method used here is taken over from the NASALIFE report [7]. First the amplitudes and mean values of individual stress tensor components over the loading cycle are computed:

$$\sigma_{ij,a} = \frac{\max\limits_t\left(\sigma_{ij}\right) - \min\limits_t\left(\sigma_{ij}\right)}{2} \qquad \sigma_{ij,m} = \frac{\max\limits_t\left(\sigma_{ij}\right) + \min\limits_t\left(\sigma_{ij}\right)}{2} \qquad (7)$$

The amplitudes of stress tensor components are processed into the equivalent stress amplitude based on the von Mises solution:

$$\sigma_a = \sqrt{\frac{1}{2}\left[\left(\sigma_{x,a} - \sigma_{y,a}\right)^2 + \left(\sigma_{y,a} - \sigma_{z,a}\right)^2 + \left(\sigma_{z,a} - \sigma_{x,a}\right)^2 + 6\left(\tau_{xy,a}^2 + \tau_{yz,a}^2 + \tau_{zx,a}^2\right)\right]}. \qquad (8)$$

The mean stress is computed in the similar way from the individual components, but the sign based on the sign of the first stress tensor invariant with the largest deviation from zero during the cycle analyzed is appended to it:

$$\sigma_m^* = \sqrt{\frac{1}{2}\left[\left(\sigma_{x,m}-\sigma_{y,m}\right)^2+\left(\sigma_{y,m}-\sigma_{z,m}\right)^2+\left(\sigma_{z,m}-\sigma_{x,m}\right)^2+6\left(\tau_{xy,m}^2+\tau_{yz,m}^2+\tau_{zx,m}^2\right)\right]}$$

$$\sigma_m = \sigma_m^* \cdot sign\left[I_{1,d}\right], \qquad (9)$$

NASALIFE report [7] recommends the use of the Walker mean stress effect described here in Eqs. (5) and (6). There are other modifications of the Manson-McKnight method as presented in NASALIFE report [7] or by Filippini et al [8]. The results of analyses on the same test set have not persuaded that these clones are substantially better if other than very specific load cases are evaluated. These modifications were not accepted into this evaluation therefore.

4.2.2 Signed von Mises stress

This solution is based on the computation of the equivalent von Mises stress for every time instant t in the cycle:

$$\sigma(t) = \sqrt{\frac{1}{2}\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]} \qquad (10)$$

Different sign corrections are defined in order to cope with the fact that the use of such a value would produce the signals with the positive sign only, i.e. with intrinsic mean stress involved. Solution accepted here is based on the instantaneous sign of the first invariant of the stress tensor:

$$\sigma^*(t) = \sigma(t) \cdot sign\left(I_1(t)\right) \qquad (11)$$

The stress amplitude and the mean stress are computed from the course of this signal. Another signing solution is e.g. used in MSC.Fatigue solver. Here the sign is derived from the sign of the principal stress with the largest deviation from zero during the load cycle. A little influence of the signing solution in comparison to tremendous scatter of result data was found, so only the method in Eq. (11) is presented here.

4.2.3 Von Mises per partes

Because of the poor quality of the NASALIFE report [7] manifested in a frequent occurrence of typing errors in the equations, the solution further described as Sines per partes in Sec. 4.2.4 was checked also without any mean stress effect. This means that only the equivalent stress amplitude computed from Eq. (8) was introduced into the comparison with the fatigue limit in Eq. (1). Though zeroizing completely the mean stress effect is inadmissible, this step allows a look into the functionality of the mean stress effects inclusions elsewhere. Because of this fact, results of this reduced method were also admitted into our comparison.

4.2.4 Sines per partes

The stress amplitude from Eq. (8) is accompanied by the mean value computed from normal mean stresses:

$$\sigma_m = \sigma_{xm} + \sigma_{ym} + \sigma_{zm} \qquad (12)$$

calculated from Eq. (7),in order to form the solution attributed to Sines and Ohgi in NASALIFE [7]:

$$\sigma_{a,eq} = \sigma_a + C \cdot \sigma_m \qquad (13)$$

NASALIFE [7] provides a proposal on C parameter to be 0.5 for uniaxial loading conditions and 1.0 for multiaxial loading conditions. Anyway, any use of so high C parameters lead to disastrous quality of results and a suitable value valid for any load condition was found to be C = 0.2. This value is used in further analyses.

4.2.5 Sines per partes & Walker MSE

This method is based on Eq. (8) and Eq. (12) noted above, but mixes them in another way, using the Walker mean stress correction reported to in Eq. (5).

4.2.6 Effective method

No reference to some historical source was found in NASALIFE report [7] as regards this method. The stress amplitude is again computed as reported in Eq. (8), but the mean stress computation follows another way. The mean stress σ^*_m from Eq. (9) is multiplied by factor two and the stress amplitude is subtracted from it:

$$\sigma_m = 2 \cdot \sigma_m^* - \sigma_a \qquad (14)$$

Then the solution follows the Walker mean stress correction described in Eq. (5).

Table I. Statistical evaluation of the mean fatigue index error. Abbreviations : nMS~no mean loads active; MS~loading including mean value; OP~out-of-phase loading; IP~in-phase loading; Ax~loading on the axial channel; To~loading on the torsion channel; noPS~no phase shift between load channels; PS<>0~non-zero phase shift between load channels;

Mean values of ΔFI in individual groups (tests)	Dang Van	Papuga PCr	Sines	Manson-McKnight	Signed von Mises	Von Mises per partes	Sines per partes	Sines per partes & Walker	Effective method
All (407)	-0.1	-0.5	-4.3	5.0	-4.5	-7.1	1.8	5.0	4.8
nMS (171)	-0.6	0.9	-12.1	4.9	0.8	4.9	4.9	4.9	4.9
nMS,OP (40)	-7.9	0.1	-20.8	8.4	-9.0	8.4	8.4	8.4	8.4
nMS,IP (131)	1.7	1.1	-9.5	3.8	3.8	3.8	3.8	3.8	3.8
MS (236)	0.1	-1.5	1.4	5.1	-8.4	-15.8	-0.5	5.1	4.7
MS,Ax (41)	-0.8	-1.9	9.2	2.3	2.2	-24.4	-4.3	2.3	6.5
MS,To (18)	-16.4	-8.5	-16.4	20.0	9.7	-5.3	-5.3	20.0	21.2
MS,Ax+Ax (36)	11.9	-4.2	17.7	-7.1	-24.2	-28.0	-2.7	-7.1	-5.3
MS,Ax+Ax,noPS (18)	14.8	-4.3	16.1	2.9	-25.5	-18.2	8.6	2.9	4.6
MS,Ax+Ax, PS<>0 (18)	9.0	-4.0	19.3	-17.2	-22.9	-37.9	-14.0	-17.2	-15.2
MS,Ax+To (114)	-3.0	0.6	-6.0	10.2	-3.2	-9.0	1.3	10.2	7.2
MS-Ax, Ax+To (52)	-0.9	-0.5	3.8	11.6	-29.1	-7.0	6.4	11.6	4.2
MS-To, Ax+To (31)	-9.6	3.0	-24.7	14.9	45.7	-6.6	-6.6	14.9	14.4
ductile (352)	-0.1	-0.7	-2.0	4.4	-6.4	-9.4	0.9	4.4	4.0
ductile,nMS (118)	-0.8	1.1	-9.6	3.6	-1.0	3.6	3.6	3.6	3.6
ductile,nMS,IP (86)	2.7	1.8	-5.9	3.3	3.3	3.3	3.3	3.3	3.3
brittle (37)	-1.5	-0.4	-28.9	13.6	12.2	10.9	10.9	13.6	14.1
brittle, nMS(35)	-1.0	-0.7	-28.3	11.7	8.7	11.7	11.7	11.7	11.7
brittle,nMS,IP (29)	-2.0	-1.9	-27.8	7.9	7.9	7.9	7.9	7.9	7.9

Table II. Statistical evaluation of the range of the fatigue index errors. Abbreviations are described by Table I.

Range of ΔFI in individual groups (tests)	Dang Van	Papuga PCr	Sines	Manson-McKnight	Signed von Mises	Von Mises per partes	Sines per partes	Sines per partes & Walker	Effective method
All	92.9	37.4	114	93.8	271.3	121.0	105.9	93.8	125.9
nMS (171)	52.7	22.7	61.6	70.6	78.4	70.6	70.6	70.6	70.6
nMS,OP (40)	52.7	22.7	49.6	70.3	78.4	70.3	70.3	70.3	70.3
nMS,IP (131)	22.3	16.5	51.9	44.8	44.8	44.8	44.8	44.8	44.8
MS (236)	92.9	37.4	113.1	93.7	271.3	85.5	79.5	93.7	125.9
MS,Ax (41)	63.8	32.6	71.4	61.6	61.6	56.4	61.1	61.6	108.2
MS,To (18)	46	16.6	46	45.5	80.6	65.4	65.4	45.5	55.0
MS,Ax+Ax (36)	57.2	32.2	37.5	50.4	115.3	49.3	68.1	50.4	54.4
MS,Ax+Ax,noPS (18)	42.7	24.5	31.5	34.3	91.2	33.0	47.2	34.3	36.8
MS,Ax+Ax, PS<>0 (18)	57.2	26.6	35.7	34.6	102.5	26.8	42.2	34.6	36.7
MS,Ax+To (114)	61.2	35.6	88.1	74.1	271.3	70.1	69.5	74.1	90.2
MS-Ax, Ax+To (52)	61.2	27.9	73.7	46.9	98.9	63.8	61.1	46.9	49.0
MS-To, Ax+To (31)	40.5	24.3	39.1	63.5	187.6	41.3	41.3	63.5	79.4
ductile (352)	92.9	37.4	113.1	70.5	271.3	85.5	79.5	70.5	125.9
ductile,nMS (118)	42.8	22.1	43.4	30.4	45.4	30.4	30.4	30.4	30.4
ductile,nMS,IP (86)	17.5	14.4	21.4	18.4	18.4	18.4	18.4	18.4	18.4
brittle (37)	42.4	17.1	38.5	69.8	104.4	76.4	76.4	69.8	80.0
brittle, nMS(35)	42.4	17.1	38.5	69.8	66.7	69.8	69.8	69.8	69.8
brittle,nMS,IP (29)	16.4	9	28.8	44.0	44.0	44.0	44.0	44.0	44.0

5 RESULTS

The results are summarized in two tables. Table I provides an analysis of the mean fatigue index error within the specified groups. To help in a quick evaluation, two margins are highlighted by colours of fonts and cells. The mean values of ΔFI more distant from zero than 5%, but lower than 10% are highlighted by the grey cell background. Values higher than 10% have black background of the cells, but the fonts are white.

Table II serves for analysing the ranges of the ΔFI factors, i.e. their max-min values. The ranges higher than 80% are marked by black cell with white font, lower values, but higher than 40%, are shown in grey cells with black fonts.

Papuga [1] uses the third parameter for the evaluation – the standard deviation. Its use here would substantially increase the demand on space, while the assets of its inclusion are doubtful. The trends are quite well represented in Tables I and II.

A note should be made on explaining the classification to ductile and brittle materials, which is used here. The ductile materials are those here, where the fatigue limit ratio of the fatigue limit in fully reversed axial loading to the fatigue limit in fully reversed torsion is higher than 1.25 but lower than 1.73. Brittle materials are categorized as those with the fatigue limit ratio below this domain, while extra-ductile materials are above this domain. The group of extra-ductile materials has very limited number of representatives in the FatLim database and the prediction results behave very similar to ductile materials for the current set of methods, so it has not been included into the analysed group reproduced here.

5.1.1 Overall statistics {All}

Results of two methods are apparently different from the others. The Signed von Mises method shows scatter in results higher than any method here. It is obvious that this type of solution is quite dangerous, though further studies hereafter show cases, where its use is safe. On the other hand, the Papuga PCr method clearly shows the advantages of the true multiaxial solution relying on the critical plane concept.

Three methods show similar scatter of results – these are the Dang Van, Manson-McKnight and Sines per partes & Walker methods. Within this rough evaluation, the quick conclusion could be that the asset of using the favourite multiaxial Dang Van solution with its computational complexity is not necessary and these simpler methods could replace it. Both simpler methods show deviations of the fatigue index error mean value close to 5%, i.e. on the safer, conservative side, which needn't to be accepted as a wrong result.

The von Mises per partes solution has the mean value of the fatigue index error shifted substantially to unsafe side. This is quite a logical result, because this method does not have any mean stress effect included. On the other hand, one could expect much larger deviation in such a case.

5.1.2 No mean stresses, in-phase loading {nMS, IP}

This is the simplest multiaxial loading, as regards the demand placed on the methods. The simplified methods are usually described as providing acceptable results for proportional loading, which is not wholly the same in meaning, because it covers also cases with the same $R=(\sigma_{min}/\sigma_{max})$ ratio on both load channels. Anyway, all the methods should be perfect in this group of results, because {nMS, IP} group is a sub-domain of proportional loading.

The results of all simplified methods are the same. It is interesting to see that these methods provide better mean values and ranges than the Sines method. Also the further separation to brittle and ductile materials uncovers interesting information. The mean value of the fatigue index error is completely different for the Sines method, which is the main reason for poor results of the Sines method in the {nMS, IP} group. On the other hand, the range of results is much higher on brittle materials than on ductile materials for all simplified methods.

So overall, all the methods provide nice results only for the ductile materials under {nMS, IP} condition, with the exception of the Sines method with a bit unsafe mean value of the fatigue index error. The simplified methods cannot be recommended for the use on brittle materials, where the Dang Van and Papuga PCr methods are the optimum choice.

5.1.3 No mean stresses, out-phase loading {nMS, OP}

There is not enough representative data for separating the materials to ductile and brittle. Anyhow, it is clear that the extension of the previous load conditions to the out-of-phase loading substantially worsens the predictive capability of the most of the methods, including the multiaxial methods, with the exception of the Papuga PCr method, which provides similar prediction statistics for IP and OP groups. Though the Sines method results in a smaller range of results for the OP condition than for IP, its mean value under OP loading is shifted to non-conservative prediction so heavily, that it is inacceptable. The positives of the multiaxial methods (namely Dang Van and Papuga PCr) are obvious anyway.

5.1.4 Uniaxial loading with mean stresses {MS, Ax} and {MS, To}

These load cases are common uniaxial loading with no trace of the load multiaxiality, so the common expectation would be that all methods can cope with them without any hesitation. It has to be noted, that the severity of loading (magnitude of the mean stresses) is much higher for the axial loading, which probably also results in smaller ranges of torsion load cases when compared to axial loading. The only exception is the signed von Mises solution, which was already marked as inacceptable in Sec. 5.1.1.

The check of the mean values of the fatigue index errors for these two groups discloses, that the methods poorly behaving for the torsion load case with the exception of the Sines per partes method. Because the coefficient weighting the effect of the mean stresses was tuned for the Sines per partes method to fit with the current group of test data, it cannot be said that its good behaviour is universal.

If the evaluation of the changes of the mean value of the fatigue index error is not so severe, the functionality of the Papuga PCr and signed von Mises could be accepted, but the latter one should be abandoned from other reasons (Sec. 5.1.1).

5.1.5 Biaxial loading with mean stresses {MS, Ax+Ax}

This type of loading is typical for pressure vessels. As noted by Papuga in [1] it would be perfect to have more data on biaxial loading without mean values, but, probably due to experimental reasons, a statistically representative group with such a property is not available. Thus the biaxial loading can be assessed only with mean stresses involved. It should be noted that the number of experiments in this group is not very big.

Only the Papuga PCr method provides acceptable results. Too conservative (Dang Van) and non-conservative (Sines) predictions limit the usage of these multixial methods. If we keep to pressure vessels, there is no phase shift involved, i.e. only the group (MS, Ax+Ax, noPS) usually refers to it. The results of the Manson-McKnight and Sines per partes & Walker method are quite good here. The nice range of the von Mises per partes method is discredited by the awfully low mean value of the fatigue index error, thus manifesting the lacking mean stress effect inclusion in this method. The results of the Effective method are also acceptable.

5.1.6 Axial and torsion loading including the mean stresses {MS, Ax+To}

Simultaneous axial and torsion loading is not well mastered by Manson-McKnight and Sines per partes & Walker methods, which tend to be too safe under such conditions. The poor results of the Signed von Mises solution were already mentioned. If the mean load is only on the axial load channel, the results of the Effective method can be qualified as good, followed by Manson-McKnight and Sines per partes & Walker with overly conservative mean value. If the mean torsion load is involved only, the Sines per partes solution is close to capabilities of the Dang Van method with the mean value of the fatigue index error even less shifted to unsafe side (but shifted substantially anyway). The outcome here is evident, only the PCr method is able to cover all load cases, a careful evaluation of the load case elsewhere is necessary.

5.1.7 Ductile and brittle materials

The group of brittle materials is quite small, but even in its limited size shows evidently that the von Mises based solutions are not the right tool to be used, if the trend of the over-conservative mean prediction value is not accepted as the right choice to keep the prediction safe. If the ductile materials are evaluated only, it is interesting to note how many methods can provide better results than the Dang Van or Sines methods. On the other hand, it should be understood, that this is a large-scale evaluation and for specific purposes (as noted above) the Dang Van method can provide substantially better results than the simplified methods.

5.1.8 Distinction between Manson-McKnight and Sines per partes & Walker methods

Anybody looking on Tables I and II can note that the results of both these methods are the same. Indeed, both criteria coincide if the negative mean stresses are removed from the analyses as was the case here. If the negative mean stresses are involved, the results of Manson-McKnight are better as can be seen in Table III.

Table III. Comparison of Manson-McKnight and Sines per partes & Walker methods, if the load cases with negative mean stresses are admitted.

		All (407)	MS,Ax (41)	MS,To (18)	MS,Ax+Ax (36)	MS,Ax+Ax,noPS (18)	MS,Ax+Ax, PS<>0 (18)	MS,Ax+To (114)	MS-Ax, Ax+To (52)	MS-To, Ax+To (31)	ductile (352)	brittle (37)
Mean values of ΔFI in individual groups (tests)	Manson-McKnight	5.3	2.0	20.0	-7.5	2.8	-17.7	11.3	13.6	14.9	4.6	13.9
	Sines per partes & Walker	6.0	2.7	20.0	-7.5	2.8	-17.7	13.4	15.5	14.9	5.2	15.9
Range of ΔFI in individual groups (tests)	Manson-McKnight	95	62	46	51	34	36	74	69	64	94	70
	Sines per partes & Walker	127	62	46	51	34	36	107	102	64	127	78

6 CONCLUSION

The evaluation presented here included 3 multiaxial and 6 simplified methods based on the von Mises equivalent stress. The intention was to analyse under which conditions these simplified methods could provide acceptable results and when they should be abandoned at all. The results show, that the domains, where some method are better to be used than the other ones and potentially, where the use of methods soluble with simple analytical tools (MS Excel) could be sufficient and no further costs on multiaxial solution are necessary, are quite limited in their size and definition.

Overall, from all the simplified methods, the Manson-McKnight solution provided the best results. Under specific conditions and large-scale evaluation (e.g. all experiments on ductile materials), it yielded better results than Dang Van and Sines methods. Elsewhere, the use of specific simplified methods can be recommended only for specific load conditions as noted above in separate sections. None of the simplified methods nevertheless provides such a predictive reliability not depending on the current load conditions as the Papuga PCr method. Results of this method show, why the multixial methods, if carefully selected, should be preferred.

7 ACKNOWLEDGEMENT

The project of the search for the right simplified solution was possible thanks to the support of the Czech Science Foundation (GACR 101/09/0904 project), which is gratefully acknowledged.

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